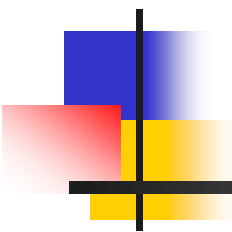


# SEA+



## **RRI** Reciprocal Radiation Integral for SEA Modeling

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InterAC



# Reciprocity Principle

- Acoustic excitation of a structure is a reciprocal quantity of its capabilities of radiating back vibrational energy in the related fluid due to the linearity of dynamical operators driving the vibroacoustic fields
- Power injected in a structural modal by a pressure field is described using the joint acceptance term  $J^2$  with  $A$  the wet area and  $P^2$  the autospectrum of applied walled pressure

$$P_{inj} = J^2 A^2 P^2$$

- Radiated Power is proportional to the radiation efficiency with  $\rho_0 c_0$  the fluid characteristic impedance and  $V^2$  the autospectrum of mean structural velocity

$$P_{rad} = \sigma_{rad} \rho_0 c_0 A V^2$$

- For a diffuse acoustic field excitation, it can be demonstrated that for an elementary structural mode of resonant radian frequency  $\omega_i$

$$J_{ii}^2(\omega, \omega_i) = \beta(k_0, A) \cdot \sigma_{irad}(\omega, \omega_i)$$

# Reciprocal Radiation Integral

- From previous relationship between joint acceptance and radiation efficiency, the latter can be directly computed from the joint acceptance i.e. by loading the structural modes by appropriate sound waves which gives the possibility to process any structural eigenshapes  $\vec{\psi}$  extracted from a FEM model and map the radiation efficiency over frequency and solid angle of radiation,  $\Omega$

$$\sigma_{rad} = \frac{k^2 J^2}{4\pi^2 A} = \alpha(k) \int_{\Omega} |P\psi_n|^2 d\Omega$$

- In the case of a structure excited by a diffuse field, from the mapping of modal radiation efficiencies, transmission coefficient and velocity response of the structure to sound waves can be explicitly calculated from the modal amplitude function  $H^2(\omega, \omega_i)$

$$\rho_i = \frac{1}{\int_A |\psi_i|^2}$$

$$\tau_d(\omega) = \frac{\Pi_{rad}(\omega)}{\Pi_{inc}(\omega)} = \frac{[\rho c]^2}{A} \frac{16\pi}{k^2} \sum_{i/\omega_i \in \Delta\omega} \frac{\sigma_{si}^2}{\rho_{si}^2} H^2(\omega, \omega_{si})$$

$$V^2(\omega) = \frac{4\pi}{Ak^2} \sum_i \frac{\sigma_i}{\rho_{si}^2} H_i^2$$



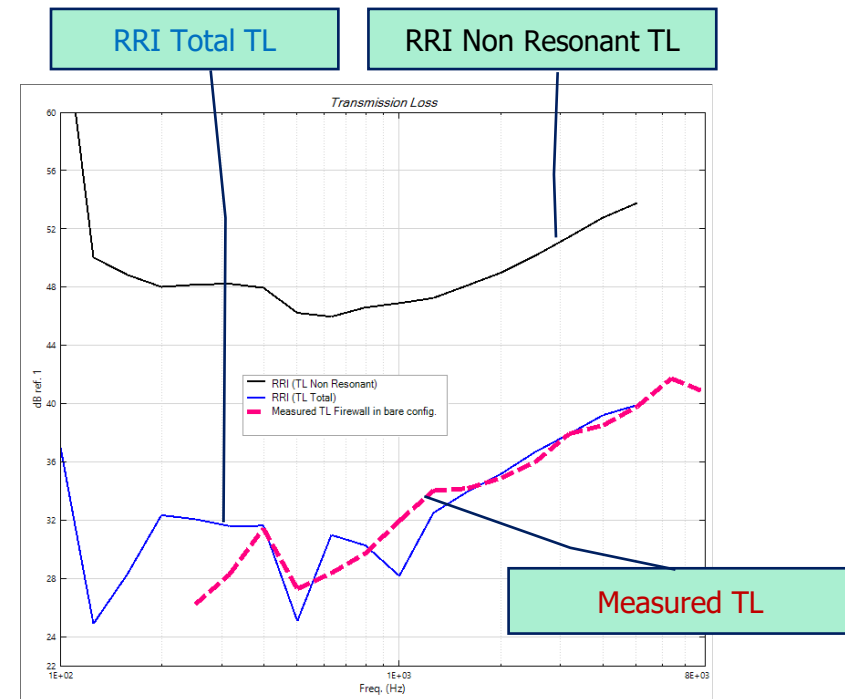
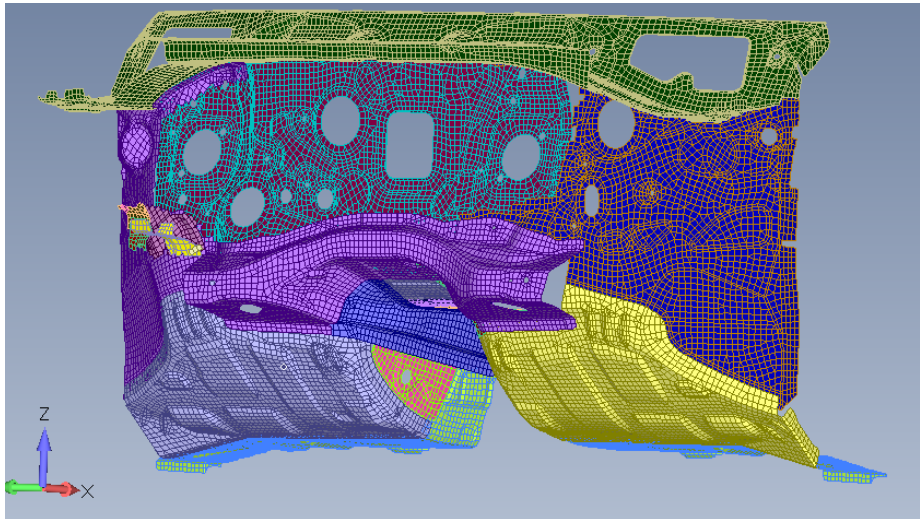
# SEA+ Implementation

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- Exporting from NASTRAN FEM model a compact group of elements and nodes with related modal amplitudes in an OP2 or PCH file
- Importing this file from SEA+ database
- Processing RRI radiation efficiency mapping from the file
- Storing the result under RRI Radiation Efficiency in database
- Post-processing for a given damping spectrum and user-defined frequency band
  - TL total, resonant and non-resonant
  - $V^2$  total and resonant
  - Modal radiation efficiency used by ASEA junctions
  - Mass (frequency)

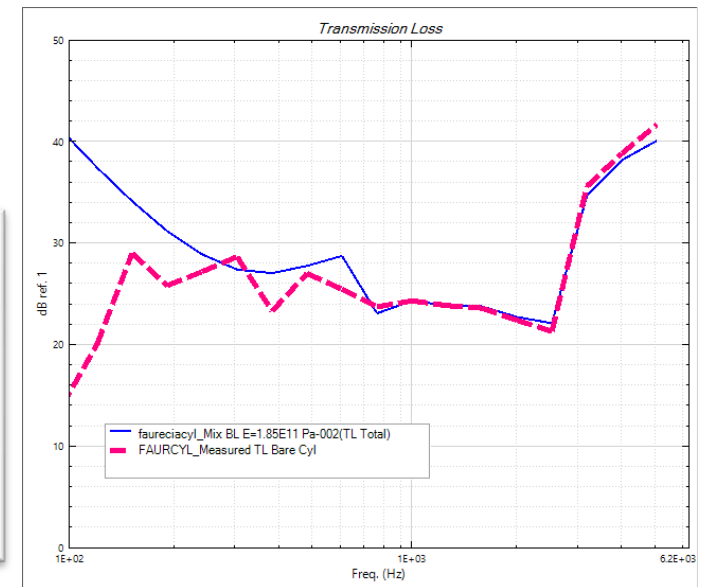
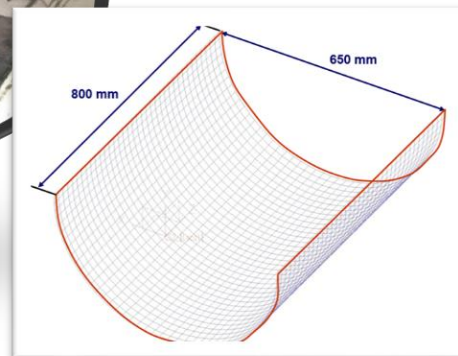
# Car Firewall TL

- OP2 Export containing modal amplitudes from NASTRAN FEM of nodes on external area of the firewall
- Processing RRI radiation efficiency mapping from the file and post-processing of TL and other related RRI parameters
- RRI TL is compared to available TL measurement in untrimmed configuration
- Good correlation found with Test with 3% DLF



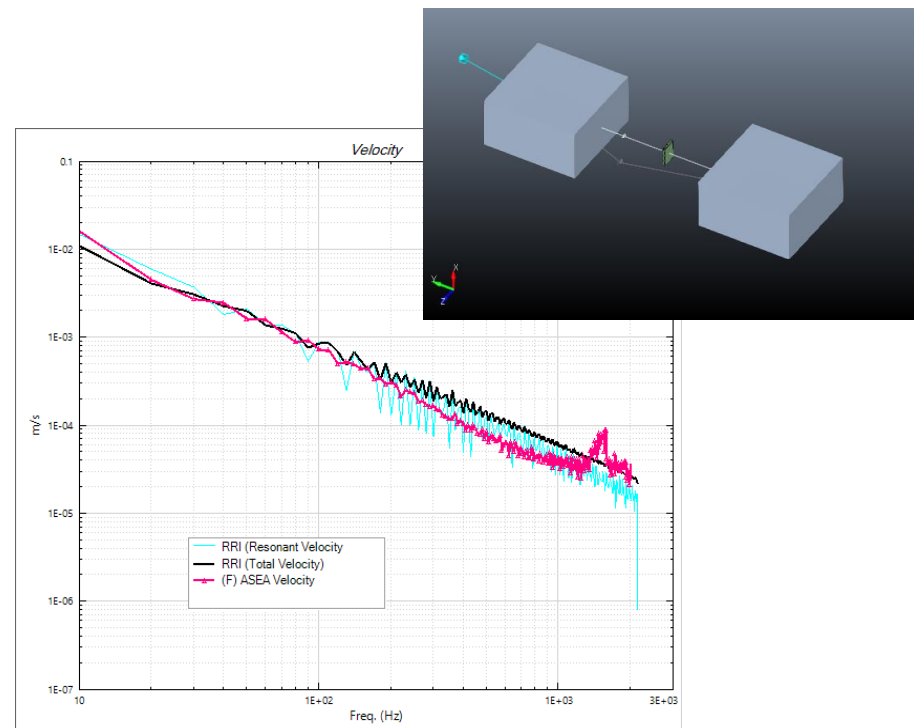
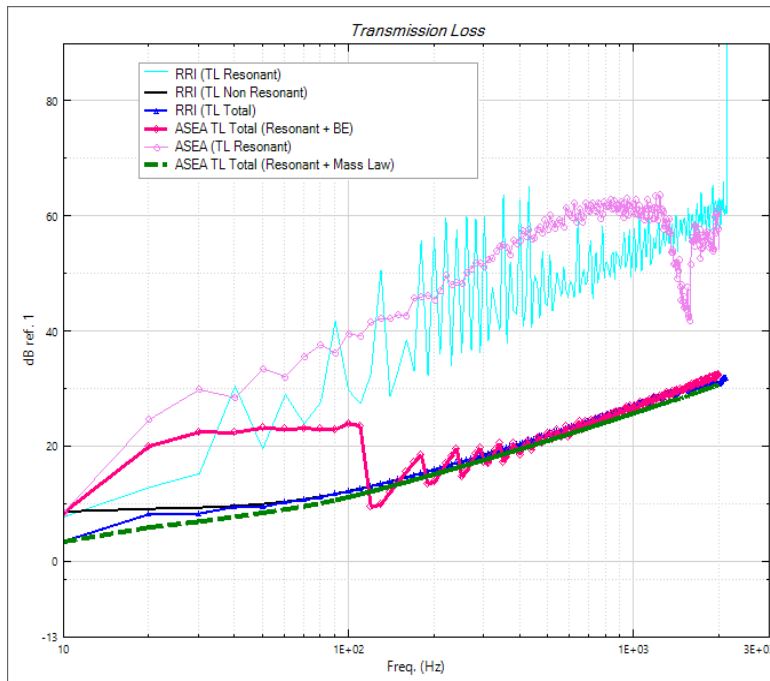
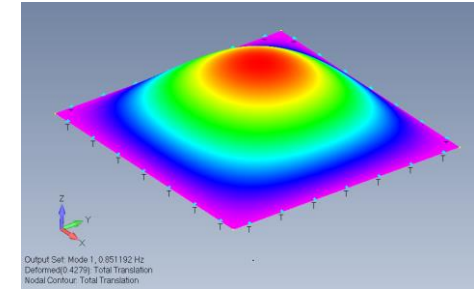
# Half-Cylinder TL

- OP2 Export containing modal amplitudes from NASTRAN FEM of nodes on external area of the firewall
- Processing RRI radiation efficiency mapping from the file and post-processing of TL and other related RRI parameters
- RRI TL is compared to available TL measurement in untrimmed configuration
- Good correlation of total RRI TL found with Test with 2% DLF. Non-resonant TL predominant below 800 Hz



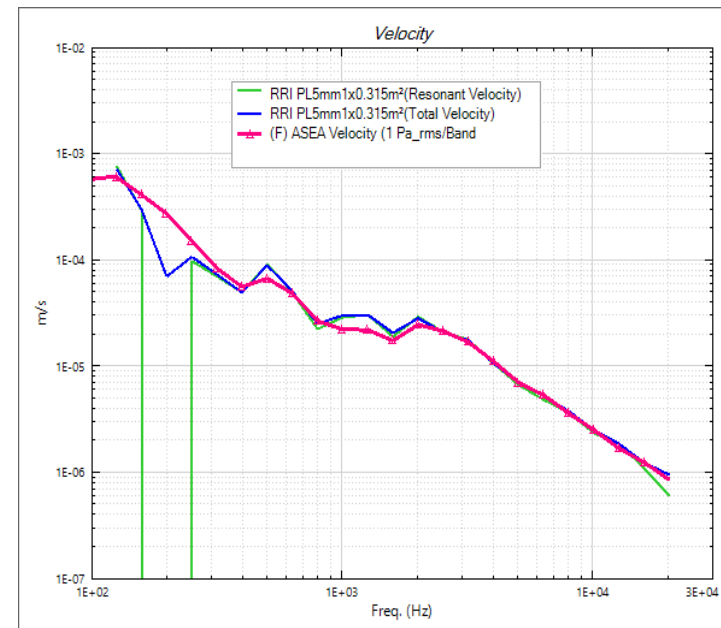
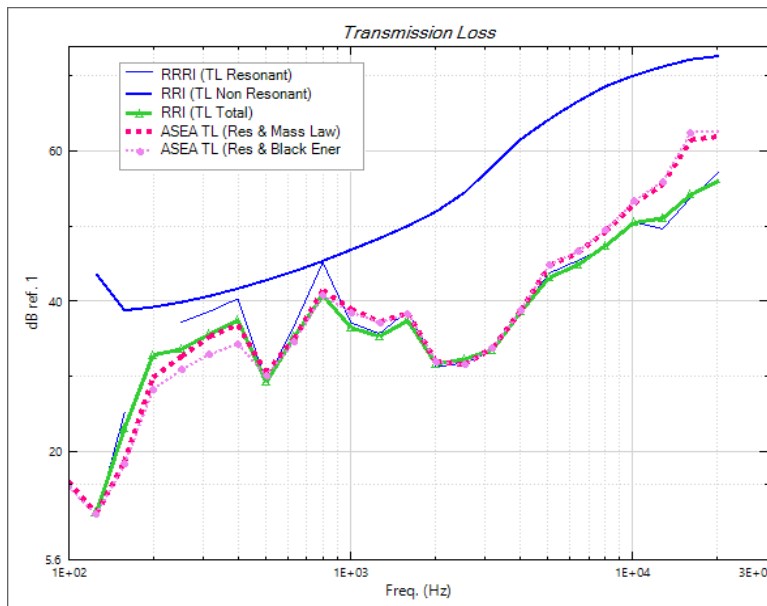
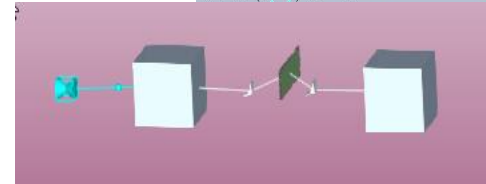
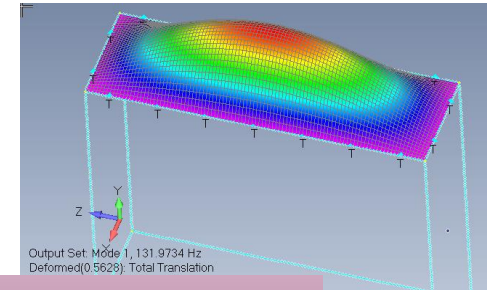
# RRI & ASEA Plate (Steel-0.7mm-2x2 m<sup>2</sup>)

- Plate is assumed simply-supported with 1% DLF
- ASEA model with same setting
- Dominant path: mass law (RRI and ASEA TL within 1 dB above 100 Hz). Good correlation of resonant TL and velocities



# RRI & ASEA Plate (Steel-5 mm-1x0.315 m<sup>2</sup>)

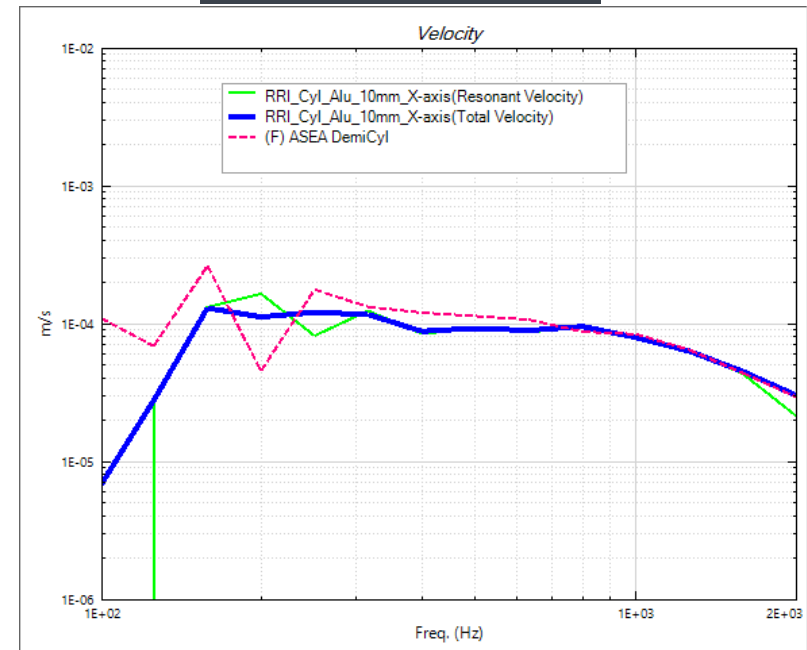
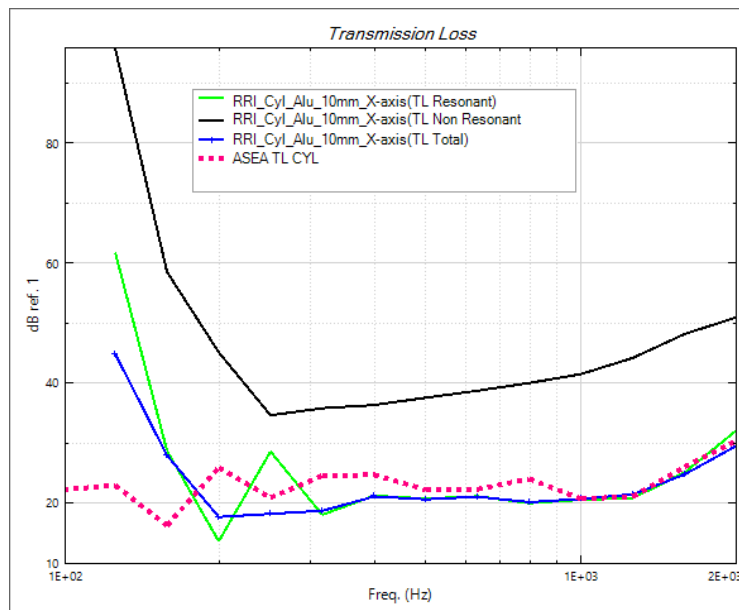
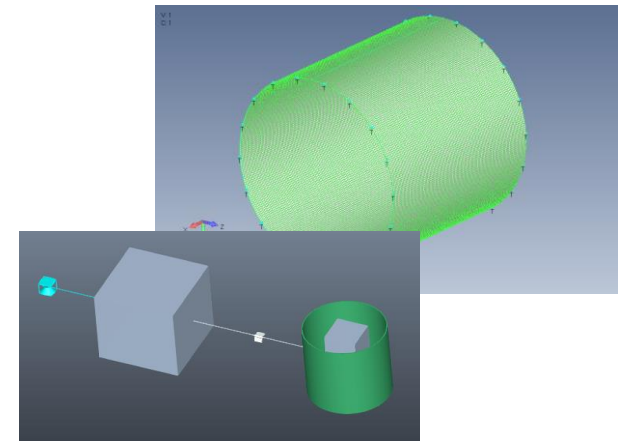
- Plate is assumed simply-supported with 1% DLF
- ASEA model with same setting
- Dominant path: Resonant (RRI and ASEA TL within 2 dB above 100 Hz). Good correlation of resonant TL and velocities





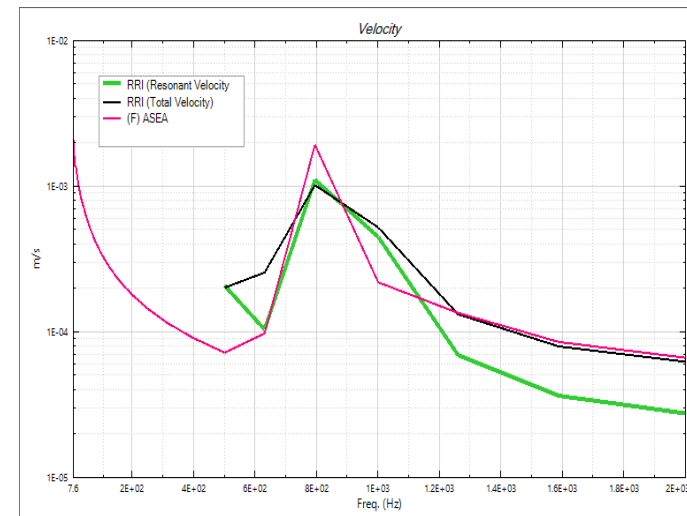
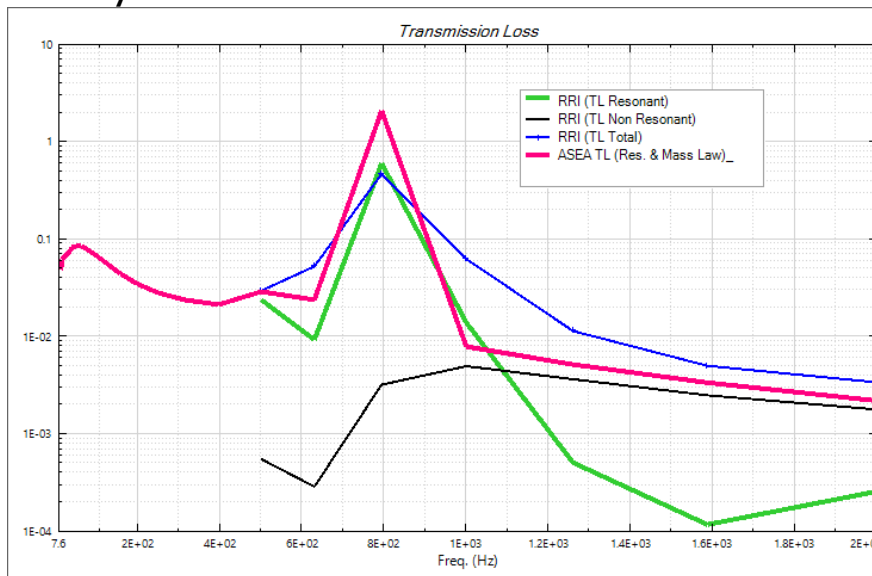
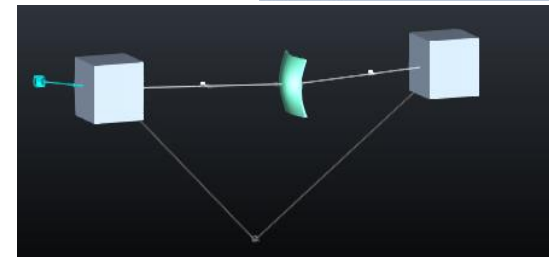
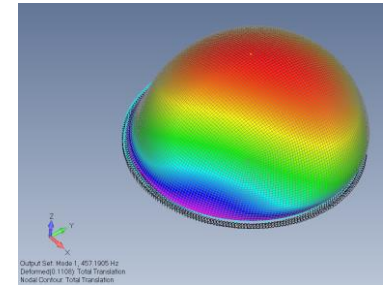
# RRI & ASEA Closed Cylinder (Alu-10 mm-2 m Length with R=1m)

- Cylinder is assumed simply-supported with 1% DLF
- ASEA model with same setting
- Dominant path: Resonant (RRI and ASEA TL within 2 dB above 200 Hz). Good correlation of resonant TL and velocities



# RRI & ASEA Sphere (Alu-1 mm-2 R=1m)

- Half-sphere is assumed simply-supported with 1% DLF
- ASEA model with same setting
- Dominant path: Resonant & Non-resonant (RRI and ASEA TL within 5 to 2 dB above 600 Hz). ASEA model more dispersed as doubly-shell dynamic operator is an approximation
- RRI allows setting up better analytical model of doubly-shell





# Conclusions

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- RRI accurately identifies radiation efficiency map of structural components from its real mode shapes and eigenfrequencies
- It allows to deliver SEA-compatible acoustic descriptors on non-homogeneous structures with a high degree of fidelity in a direct usable format such as  $TL(f)$  or in SEA band-format for inserting RRI spectra as user-defined in SEA connections (cavity-to-cavity and structure-to-cavity)
- It delivers also the mean velocity under diffuse acoustic loading which enables easy identification of damping loss factor of the tested structure
- Amount of non-resonant and resonant transmission is provided, which is the solution to the main SEA acoustic modeling difficulty: calculation of non-resonant transmission in a non-homogeneous subsystem